

# Experimental Sharing of Nonlocality among Multiple Observers with One Entangled Pair via Optimal Weak Measurements

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Bell nonlocality plays a fundamental role in quantum theory. Numerous tests of the Bell inequality have been reported since the ground-breaking discovery of the Bell theorem. Up to now, however, most discussions of the Bell scenario have focused on a single pair of entangled particles distributed to only two separated observers. Recently, it has been shown surprisingly that multiple observers can share the nonlocality present in a single particle from an entangled pair using the method of weak measurements [Phys. Rev. Lett. **114**, 250401 (2015)]. Here we report an observation of double CHSH-Bell inequality violations for a single pair of entangled photons with strength continuous-tunable optimal weak measurements in photonic system for the first time. Our results shed new light on the interplay between nonlocality and quantum measurements and our design of weak measurement protocol may also be significant for important applications such as unbounded randomness certification and quantum steering.

**Introduction.** Nonlocality, which was first pointed by Einstein, Podolsky and Rosen (EPR) [1], plays a fundamental role in quantum theory. It has been intensively investigated since the ground-breaking discovery of Bell theorem by John Bell in 1964 [2]. Bell theorem states that any local-realistic theory can not reproduce all the predictions of quantum theory and gives an experimental testable inequality [3] that later improved by Clauser, Horne, Shimony and Holt (CHSH) [4]. Numerous tests of CHSH-Bell inequality have been realized in various quantum systems [5–13] and strong loophole-free Bell tests have been reported recently [14–16].

To date, however, most discussions of Bell scenario focus on one pair of entangled particles distributed to only two separated observers Alice and Bob [17]. It is thus a novel and fundamental question whether or not multiple observers can share the nonlocality present in a single particle from an entangled pair. Using the concept of weak measurements, Silva *et al.* give a surprising positive answer to above question and show a marvelous physical fact that measurement disturbance and information gain of a single system are closely related to nonlocality distribution among multiple observers in one entangled pair [18].

In this letter, we report an experimental realization of sharing nonlocality among multiple observers with strength continuous-tunable optimal weak measurements in photonic system. The realization of sharing nonlocality is certified by the observed double violations of CHSH-Bell inequality with one pair of entangled photons. Our results not only shed new light on the interplay between

nonlocality and quantum measurements but also could be found significant applications such as in unbounded randomness certification [19, 20] and quantum steering [21, 22].

**Disturbance and information gain in weak measurements.** As one of the foundations of quantum theory, the measurement postulate states that upon measurement, a quantum system will collapse into one of its eigenstates, with the probability determined by the Born rule. While this type of strong measurement, which is projective and irreversible, obtains the maximum information about a system, it also completely destroys the system after the measurement. Weak measurements [23], however, can be used to extract less information about a system with smaller disturbance, and, over the past decades, have been shown to be a powerful method in signal amplification [24–26], state tomography [27, 28] and in solving quantum paradoxes [29]. In contrast to strong projective measurements, weak measurements are non-destructive

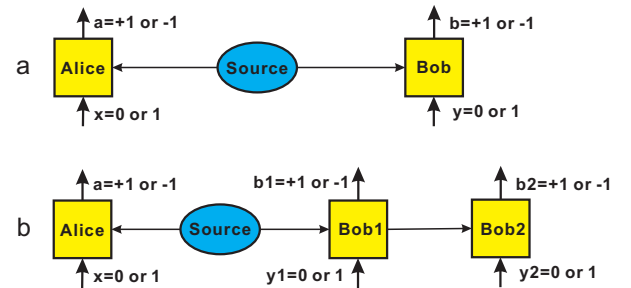


FIG. 1. **Bell test.** a. Typical Bell scenario in which one pair of entangled particles is distributed to only two observers: Alice and Bob. b. Modified Bell scenario in which Bob1 and Bob2 access the same single particle from the entangled pair with Bob1 performs a weak measurement.

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and can retain some original properties of the measured system, e.g., coherence and entanglement. Because the entanglement is not completely destroyed by weak measurements, a particle that has been measured with intermediate strength can still be entangled with other particles, and therefore, shared nonlocality among multiple observers is possible.

Consider a von Neumann-type measurement [30] on a spin-1/2 particle that is in the superposition state  $|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$  with  $|\alpha|^2 + |\beta|^2 = 1$  where  $|\uparrow\rangle$  ( $|\downarrow\rangle$ ) denotes the spin up (down) state. After the measurement, the spin state is entangled with the pointer's state, i.e.,  $|\psi\rangle \otimes |\phi\rangle \rightarrow \alpha|\uparrow\rangle \otimes |\phi_\uparrow\rangle + \beta|\downarrow\rangle \otimes |\phi_\downarrow\rangle$ , where  $|\phi\rangle$  is the initial state of the pointer and  $|\phi_\uparrow\rangle$  ( $|\phi_\downarrow\rangle$ ) indicates the measurement results of spin up (down). By tracing out the state of the pointer, the spin state becomes

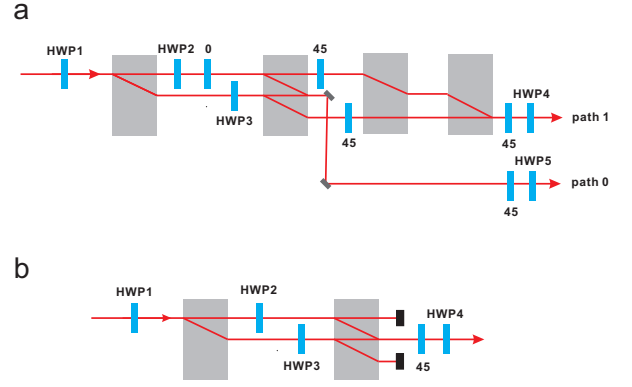
$$\rho = F\rho_0 + (1-F)(\pi_\uparrow\rho_0\pi_\uparrow + \pi_\downarrow\rho_0\pi_\downarrow), \quad (1)$$

where  $\rho_0 = |\psi\rangle\langle\psi|$ ,  $\pi_\uparrow = |\uparrow\rangle\langle\uparrow|$ ,  $\pi_\downarrow = |\downarrow\rangle\langle\downarrow|$  and  $F = \langle\phi_\downarrow|\phi_\uparrow\rangle$ . The quantity  $F$  is called the measurement quality factor because it measures the disturbance of the measurement [18]. If  $F = 0$ , the spin state is reduced to a completely decoherent state in the measurement eigenbasis, representing a strong measurement; otherwise, if  $F = 1$ , there is no measurement at all. For remaining case, where  $F \in (0, 1)$ , represents measurements with intermediate strength corresponding to weak measurements.

Another important quantity associated with weak measurements is the information gain  $G$  that is determined by the precision of the measurement [18]. In the case of strong measurements, the probability of obtaining the outcome  $+1$  ( $-1$ ) that corresponds to spin eigenstate  $|\uparrow\rangle$  ( $|\downarrow\rangle$ ) can be calculated by the Born rule  $P(+1) = \text{Tr}(\pi_\uparrow\rho_0)$  ( $P(-1) = \text{Tr}(\pi_\downarrow\rho_0)$ ). However, the non-orthogonality of the pointer states  $\langle\phi_\uparrow|\phi_\downarrow\rangle \neq 0$  in weak measurements results in ambiguous outcomes. An observer who performs a weak measurement must choose a complete orthogonal set of pointer states  $\{|\phi_{+1}\rangle, |\phi_{-1}\rangle\}$  as reading states to define the outcomes  $\{+1, -1\}$  corresponding to the spin eigenstates  $\{|\uparrow\rangle, |\downarrow\rangle\}$ . The probabilities of the outcome  $\pm 1$  in weak measurements then become  $P(\pm 1) = \text{Tr}(\pi_\uparrow\rho_0)|\langle\phi_{\pm 1}|\phi_\uparrow\rangle|^2 + \text{Tr}(\pi_\downarrow\rho_0)|\langle\phi_{\pm 1}|\phi_\downarrow\rangle|^2$ . Here,  $|\langle\phi_{+1}|\phi_\uparrow\rangle|^2$  and  $|\langle\phi_{-1}|\phi_\downarrow\rangle|^2$  correspond to the probabilities of obtaining the correct outcomes while  $|\langle\phi_{-1}|\phi_\uparrow\rangle|^2$  and  $|\langle\phi_{+1}|\phi_\downarrow\rangle|^2$  correspond to the probabilities of the wrong outcomes. For simplicity, we consider the case of symmetric ambiguousness in which  $|\langle\phi_{+1}|\phi_\uparrow\rangle|^2 = |\langle\phi_{-1}|\phi_\downarrow\rangle|^2$  and  $|\langle\phi_{-1}|\phi_\uparrow\rangle|^2 = |\langle\phi_{+1}|\phi_\downarrow\rangle|^2$ ; here, the probabilities of the outcomes can be reformulated as

$$P(\pm 1) = G \cdot \frac{1}{2}[1 \pm \text{Tr}(\sigma\rho_0)] + (1-G) \cdot \frac{1}{2}, \quad (2)$$

where  $\sigma = \pi_\uparrow - \pi_\downarrow$  defines the spin observable and  $G = 1 - |\langle\phi_{-1}|\phi_\uparrow\rangle|^2 - |\langle\phi_{+1}|\phi_\downarrow\rangle|^2$  represents the precision



**FIG. 2. Optimal weak measurements realized in photonic system.** **a:** HWP2 and HWP3 are rotated at  $\theta/2$  and  $\pi/4 - \theta/2$  degree determining the strength of measurement  $F = \sin 2\theta$ . Photons with vertical polarization state  $|V\rangle$  transmit calcite beam displacer (BD) without change of its path while photons with horizontal polarization state  $|H\rangle$  suffer a shift away from its original path. HWP1, HWP4 and HWP5 are rotated at the same degree  $\varphi/2$  to realize weak measurement of polarization observable  $\sigma_\varphi = |\varphi\rangle\langle\varphi| - |\varphi^\perp\rangle\langle\varphi^\perp|$ . The measurement outcome  $+1$  ( $-1$ ) is encoded in path 0 (1) separately. **b:** The setup, used in actual experiment, realizes same optimal weak measurement as shown above. The only difference is that specific outcome  $+1$  ( $-1$ ) can be selected by rotating HWP1 and HWP4. In the measurement of observation  $\sigma_\varphi$  with HWP1 and HWP4 rotated at  $\varphi/2$  degree, outcome  $+1$  is obtained when photons comes out of the setup and outcome  $-1$  is obtained when HWP1 and HWP4 rotated at  $\varphi/2 + \pi/4$ . Note that measurement outcome values are extracted in the final coincidence detection.

of the measurement (See more details in Supplemental Material, Part A). The quality factor  $F$  and the precision  $G$  are determined solely by the pointer states and satisfy the trade-off relation  $F^2 + G^2 \leq 1$  [18]. A weak measurement is optimal if  $F^2 + G^2 = 1$  is satisfied.

*Modified Bell test with weak measurements.* In a typical Bell test scenario, one pair of entangled spin-1/2 particles is distributed between two separated observers, Alice and Bob (Fig. 1a), who each receive a binary input  $x, y \in \{0, 1\}$  and subsequently give a binary output  $a, b \in \{1, -1\}$ . For each input  $x$  ( $y$ ), Alice (Bob) performs a strong projective measurement of her (his) spin along a specific direction and obtains the outcome  $a$  ( $b$ ). The scenario is characterized by a joint probability distribution  $P(ab|xy)$  of obtaining outcomes  $a$  and  $b$ , conditioned on measurement inputs  $x$  for Alice and  $y$  for Bob. The fixed measurement inputs  $x$  and  $y$  defines the correlations  $C_{(x,y)} = \sum_{a,b} abP(ab|xy)$ . The CHSH-Bell test is focused on the so-called  $S$  value defined by the combination of correlations

$$S = |C_{(0,0)} + C_{(0,1)} + C_{(1,0)} - C_{(1,1)}|. \quad (3)$$

While  $S \leq 2$  in any local hidden variable theory [4], quantum theory gives a more relaxed bound of  $2\sqrt{2}$  [31].

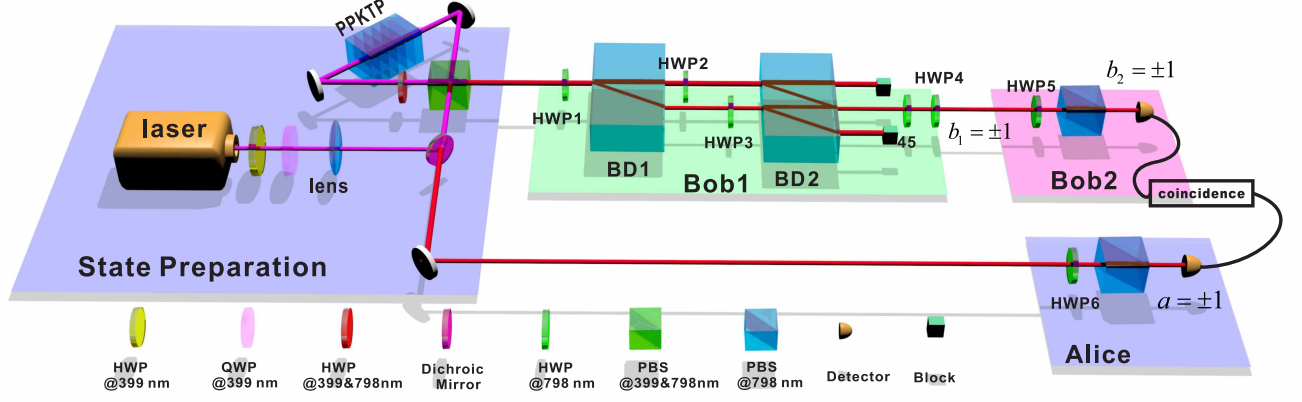


FIG. 3. **Measurement setup.** Polarization-entangled pairs of photons are produced by pumping a type-II apodized periodically poled potassium titanyl phosphate (PPKTP) crystal placed in the middle of a Sagnac-loop interferometer with dimensions of  $1\text{mm} \times 2\text{mm} \times 20\text{mm}$  and with end faces with anti-reflective coating at wavelengths of 399 nm and 798 nm. The photon emitted to Alice is measured via a combination of HWP6 and PBS. The green area shows the weak measurement setup of Bob1. During the experiment, HWP2, HWP3 are rotated by  $\theta/2, \pi/4 - \theta/2$  according to the experimental requirement. HWP1 is used for Bob1's measurement, and HWP4 is rotated by the same angle as HWP1 to transform the photons polarization state back to the measurement basis after the photon passes through two beam displacers (BDs). The photon passing through HWP4 is then sent to Bob2 for a strong projective measurement with HWP5 and PBS. In the final stage, two-photon coincidences at 6s are recorded by avalanche photodiode single-photon detectors and a coincidence counter (ID800).

Here, we consider a new Bell scenario in which there are two observers Bob1 and Bob2 access to the same one-half of the entangled state of spin-1/2 particles (Fig. 1b). Alice, Bob1 and Bob2 each receive a binary input  $x, y_1, y_2 \in \{0, 1\}$  and subsequently provide a binary output  $a, b_1, b_2 \in \{1, -1\}$ . For each input  $y_1$ , Bob1 performs a weak measurement of his spin along a specific direction while Alice and Bob2 perform a strong projective measurement for their input  $x$  and  $y_2$ . With the outcome  $b_1$ , Bob1 sends the measured spin particle to Bob2 who is totally ignorant of the existence of Bob1. The scenario is now characterized by joint conditional probabilities  $P(ab_1b_2|x y_1 y_2)$ , and an incisive question is raised whether Bob1 and Bob2 can both share nonlocality with Alice. The answer is surprisingly positive that the statistics of both Alice-Bob1 and Alice-Bob2 can indeed violate the CHSH-Bell inequality simultaneously [18].

The weak measurements quantities  $G$  and  $F$ , respectively, determine the  $S$  values of Alice-Bob1 and Alice-Bob2 in the new Bell scenario. In the case that the Tsirelson's bound  $2\sqrt{2}$  of the CHSH-Bell inequality can be attained, the calculation gives (See more details in Supplemental Material, Part B)

$$S_{A-B1} = 2\sqrt{2}G, S_{A-B2} = \sqrt{2}(1 + F). \quad (4)$$

*Realization of optimal weak measurements in photonic system.* To observe significant double violations of the CHSH-Bell inequality, the realization of optimal weak measurements is a key and necessary requirement. In the original scheme proposed in Ref. [18], the continuous

pointer is used in which pointers with Gaussian distribution only achieve sub-optimal measurement. Here, we propose and realize optimal weak measurements in photonic system by using discrete pointers i.e., path degree of freedom of photons instead of continuous pointers. It should be noted here that whether or not pointers are continuous or discrete do not change any results discussed above.

Before illustration of the realization, it should be emphasized first that weak measurements are mathematically equivalent to positive operator valued measures (POVMs) formalism [32] and this becomes our basis of experimental design. For the spin system discussed above, if Bob1 performs weak measurement and obtains outcome  $\pm 1$ , the states of measured system will accordingly collapse into

$$|\Psi_{\pm 1}\rangle_s = \alpha\langle\phi_{\pm 1}|\phi_{\uparrow}\rangle|\uparrow\rangle + \beta\langle\phi_{\pm 1}|\phi_{\downarrow}\rangle|\downarrow\rangle \quad (5)$$

with probability  $P(\pm 1) = \text{Tr}(|\Psi_{\pm 1}\rangle_s\langle\Psi_{\pm 1}|)$ . The weak measurements of Bob1 is actually to realize a two-outcome POVMs with Kraus operators [33]

$$M_{\pm 1} = \langle\phi_{\pm 1}|\phi_{\uparrow}\rangle|\uparrow\rangle\langle\uparrow| + \langle\phi_{\pm 1}|\phi_{\downarrow}\rangle|\downarrow\rangle\langle\downarrow| \quad (6)$$

corresponding to outcome  $\pm 1$ .

In our realization of weak measurements of Bob1 with photonic elements as shown in Fig. 2a, the measured photons are in polarization state and the path degree of freedom of photons is used as pointer. In order to perform weak measurements in specific polarization basis  $\{|\varphi\rangle, |\varphi^{\perp}\rangle\}$  with defined observable  $\sigma_{\varphi} =$

$|\varphi\rangle\langle\varphi| - |\varphi^\perp\rangle\langle\varphi^\perp|$ , we first transform the measured basis  $\{|\varphi\rangle, |\varphi^\perp\rangle\}$  to  $\{|H\rangle, |V\rangle\}$  via half wave plate (HWP1), then realize weak measurement of observable  $\sigma_H = |H\rangle\langle H| - |V\rangle\langle V|$  via optical elements between HWP1 and HWP4, HWP5 and finally transform back to  $\{|\varphi\rangle, |\varphi^\perp\rangle\}$  basis via HWP4 and HWP5. HWP1, HWP4 and HWP5 are rotated by the same angle  $\varphi/2$ .

The key part of our setup is the realization of weak measurements of observable  $\sigma_H$  and this is achieved by interference between calcite beam displacers (BDs) (Fig. 2 or Fig. 3). Consider photons with polarization state  $|\Phi\rangle = \alpha|H\rangle + \beta|V\rangle$  to be measured, after interaction, the composite state of photons becomes  $|\psi\rangle = \alpha|H\rangle|\phi_H\rangle + \beta|V\rangle|\phi_V\rangle$  with  $|\phi_H\rangle$  ( $|\phi_V\rangle$ ) is the corresponding pointer state. The reading states  $\{|\phi_{+1}\rangle, |\phi_{-1}\rangle\}$  in our realization are chosen as states of two separated paths 0 and 1 (see Fig. 2a) denoted by  $|0\rangle$  and  $|1\rangle$ . By rotating HWP2 and HWP3 between BDs at  $\theta/2$  and  $\pi/4 - \theta/2$  degrees respectively, the pointer states become

$$\begin{aligned} |\phi_H\rangle &= \cos\theta|0\rangle + \sin\theta|1\rangle, \\ |\phi_V\rangle &= \sin\theta|0\rangle + \cos\theta|1\rangle \end{aligned} \quad (7)$$

with  $0 \leq \theta \leq \pi/2$ . The quality factor and information gain in our case are  $F = \langle\phi_H|\phi_V\rangle = \sin 2\theta$  and  $G = 1 - |\langle 1|\phi_H\rangle|^2 - |\langle 0|\phi_V\rangle|^2 = \cos 2\theta$ . The condition of optimal weak measurements  $F^2 + G^2 = 1$  is satisfied.

In practical experiment, we use the setup shown in Fig. 2b instead of that shown in Fig. 2a. The setup shown in Fig. 2b can realize the same optimal weak measurements as that in Fig. 2a and the only difference is that specific outcome can be selected by rotating HWP1 and HWP4. When Bob1 performs weak measurement of observable  $\sigma_\varphi$  with HWP1 and HWP4 rotated at  $\varphi/2$  (or  $\pi/4 - \varphi/2$ ) degree, photons comes out of setup have state  $|\Psi_{+1}\rangle = M_{+1}|\Phi\rangle$  (or  $|\Psi_{-1}\rangle = M_{-1}|\Phi\rangle$ ) corresponding to outcome +1 (or -1). Here,  $M_{+1} = \cos\theta|\varphi\rangle\langle\varphi| - \sin\theta|\varphi^\perp\rangle\langle\varphi^\perp|$ ,  $M_{-1} = \sin\theta|\varphi\rangle\langle\varphi| - \cos\theta|\varphi^\perp\rangle\langle\varphi^\perp|$  are Kraus operators and Bob1 extracts his measurement outcomes by final coincidence detection given that the rotation angles of HWP1 and HWP4 are known to him.

*Experimental observation of double Bell inequality violations.* In our Bell test experiment (Fig. 3), polarization-entangled pairs of photons in state  $(|H\rangle|V\rangle - |V\rangle|H\rangle)/\sqrt{2}$  are generated by pumping a type-II apodized periodically poled potassium titanyl phosphate (PPKTP) crystal to produce photon pairs at a wavelength of 798nm. A 4.5mW pump laser centred at a wavelength of 399nm is produced by a Moglabs ECD004 laser, and a PPKTP crystal is embedded in the middle of a Sagnac interferometer to ensure the production of high-quality, high-brightness entangled pair [34, 35]. The maximum coincidence counts rates in the horizontal/vertical basis are approximately 3,200/s. The visibility of coincidence detection for the maximally entangled state is measured to be  $0.997 \pm 0.006$  in the horizontal/vertical

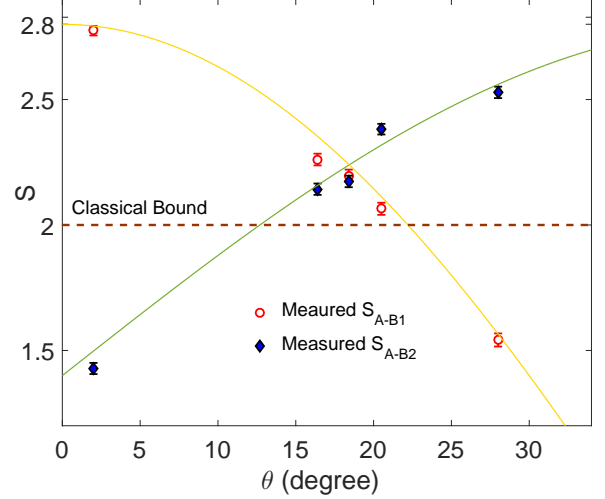


FIG. 4. **Experimental results.** The yellow and green curves represent the theoretical predictions for  $S_{A-B1}$  and  $S_{A-B2}$ , respectively, while the red circles and blue rhombus indicate the practical measured results for  $S_{A-B1}$  and  $S_{A-B2}$ , corresponding  $\theta = \{4^\circ, 16.4^\circ, 18.4^\circ, 20.5^\circ, 28^\circ\}$ . Double violations are observed in  $\theta = \{16.4^\circ, 18.4^\circ, 20.5^\circ\}$  with approximately 10 standard deviations. The error bars are calculated according to Poissonian counting statistics.

polarization basis  $\{|H\rangle, |V\rangle\}$  and  $0.993 \pm 0.008$  in the diagonal/antidiagonal polarization basis  $\{(|H\rangle \pm |V\rangle)/\sqrt{2}\}$ , achieved by rotating the polarization analyzers for two photons.

Alice, Bob1 and Bob2 each have two measurement choices, and for each choice, two trials are needed, corresponding to two different outcomes. For each fixed  $\theta$ , which determines the strength of the weak measurement  $F = \sin 2\theta$ , we implemented 64 trials for calculating  $S_{A-B1}$  and  $S_{A-B2}$ . To ensure that the Tsirelson's bound  $2\sqrt{2}$  can be approached, Alice chooses a measurement along direction  $Z$  or  $X$ , while Bobs choose a measurements along  $(-Z + X)/\sqrt{2}$  or  $-(Z + X)/\sqrt{2}$  direction. In this experiment, HWP6 is set at  $(0^\circ, 45^\circ)$  or  $(22.5^\circ, 67.5^\circ)$ , corresponding to Alice's measurement along the  $Z$  or  $X$  direction, while HWP1 and HWP5, representing measurements of Bob1 and Bob2, are set at  $(-11.25^\circ, 33.75^\circ)$  or  $(11.25^\circ, 56.25^\circ)$ , corresponding to the  $(-Z + X)/\sqrt{2}$  or  $-(Z + X)/\sqrt{2}$  direction, respectively. For instance, if HWP1, HWP4 and HWP5 are rotated at  $-11.25^\circ$  and HWP6 is fixed at  $0^\circ$ , the three-variable joint conditional probability  $P[a = 1, b_1 = 1, b_2 = 1 | x = Z, y_1 = (-Z + X)/\sqrt{2}, y_2 = (-Z + X)/\sqrt{2}]$  is obtained by final coincidence detection. The other joint conditional probabilities can be detected via similar various combination of HWP1, HWP4, HWP5 and HWP6.

Five different angles  $\theta = \{4^\circ, 16.4^\circ, 18.4^\circ, 20.5^\circ, 28^\circ\}$  are chosen for which the values of  $\theta = \{16.4^\circ, 18.4^\circ, 20.5^\circ\}$  are located in the region in



which double violations are predicted to be observed. In particular, the balanced double violations  $S_{A-B1} = S_{A-B2} = 2.26$  are present under optimal weak measurements when  $F = 0.6$ , corresponding to  $\theta = 18.4^\circ$ . Our final results are shown in Fig. 4, where double violations are clearly displayed at  $\theta = \{16.4^\circ, 18.4^\circ, 20.5^\circ\}$  with approximately 10 standard deviations. Considering the possible statistical error, systematic error and imperfection of our apparatus, these experimental results fit well within the theoretical predictions.

*Discussion and conclusion.* In conclusion, we have observed double violations of the Bell inequality for the entangled state of a photon pair by using a strength continuous-tunable optimal weak measurement setup. Our experimental results verify the nonlocality distribution among multiple observers and shed new light on our understanding of the fascinating properties of nonlocality and quantum measurements. The weak measurement technique used herein can find significant applications in unbounded randomness certification [19, 20], which is a valuable resource applied from quantum cryptography [36, 37] and quantum gambling [38, 39] to quantum simulation [40]. Here, the  $S$  value of the correlation between Alice and Bob2 is determined by the quality factor of Bob1's weak measurement, implying that Bob1 can control the nonlocal correlation of Alice and Bob2 by manipulating the strength of his measurement. This result provides tremendous motivation for the further quantum steering research [21, 22].

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